

WAVE RUNUP ON SMOOTH AND ROCK SLOPES OF COASTAL STRUCTURES

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ABSTRACT: Knowledge on wave-runup levels is important for a proper design of the crest height of coastal structures. An overall view of the literature supports the assertion that smooth slopes cause the highest possible runup levels. Therefore, various runup data on gentle smooth slopes of coastal structures in a large flume are presented, and a final design formula is given. Then, available data of runup on smooth and rock slopes are compared. The main part of this paper deals with runup on rock slopes, including revetments and breakwater structures. The slopes range from 1:1.5 to 1:4. About 250 tests have been performed, testing stability of armor layers, during which runup was simultaneously measured. First, a qualitative analysis is given on the influence of various parameters on runup. The final analysis results in two sets of design formulas. One set yields formulas for the assessment of various runup levels as a function of the surf similarity or breaker parameter. The other set presents the probability distribution of runup on rock slopes, written as a Weibull distribution.

INTRODUCTION

Wave runup is one of the most important factors affecting the design of coastal structures exposed to wave attack. Generally, revetments and sea dikes are designed in such a way that little or no wave runup overtops the structure. In the Netherlands, the crest height design of most sea dikes is based on a 2% exceedance level of the expected runup.

However, breakwaters and offshore rubble-mound structures are normally designed for more than 2% overtopping. Allowable overtopping there normally varies between 5% and 40% of the waves reaching the crest. Knowledge of runup levels on armored slopes, therefore, is useful in assessment of the crest height of these structures.

This paper only discusses irregular wave runup. Reviews on wave runup have been written by various researchers. Among other works important documents are "Wave Run-up and Overtopping" (1974), Battjes (1974), Ahrens (1981), and Allsop et al. (1985).

WAVE RUNUP ON SMOOTH SLOPES

A well-known formula for the 2% runup level on smooth slopes is the aged Delft formula given by Wassing (1942), which was used in the Netherlands for many years

$$Ru_{2\%} = 8H_s \tan \alpha \dots\dots\dots (1)$$

in which $Ru_{2\%}$ = the runup level exceeded by 2% of the runup heights (defined by an upcrossing method); H_s = the significant wave height at the

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toe of the structure; and α = the structure slope angle. This simple formula is valid for a wave steepness of about 0.05, and for gentle structure slopes with $\cot \alpha \geq 3$. Battjes (1974) modified the Hunt formula for the 2% runup level, including the wave steepness. This can be written as

$$\frac{Ru_{2\%}}{H_s} = C \xi_m = \frac{C \tan \alpha}{\sqrt{s_m}} = \frac{C \tan \alpha}{\sqrt{\frac{2\pi H_s}{g T_m^2}}} \dots \dots \dots (2)$$

in which ξ_m = the surf similarity parameter or breaker parameter based on the mean wave period T_m ; s_m = the wave steepness; g = the gravitational acceleration; and C = a constant. Limits for the coefficient C are 1.49 for fully developed seas and 1.87 for very young seas. Grüne (1982) found values of C between 1.33 and 2.86, based on prototype measurements. Eq. (2) is only valid for plunging waves, which means gentle structure slopes with ξ_m -values smaller than about 2, but limited to coastal structures. This means that a lower limit for (2) is approximately $\xi_m = 0.5$. Because the peak period T_p is often used instead of the mean period T_m , (2) can be rewritten, assuming a T_p/T_m ratio of about 1.1–1.2

$$\frac{Ru_{2\%}}{H_s} = C \xi_p \dots \dots \dots (3)$$

in which ξ_p = the breaker parameter, using T_p instead of T_m [see (2)], with C between 1.3 and 1.7 according to Battjes (1974) and 1.2 and 2.4 according to Grüne (1982), and ξ_p limited to $\xi_p < 2$. Ahrens (1981) described test results on several smooth slopes with $\cot \alpha = 1, 1.5, 2, 2.5, 3$, and 4. For $\xi_p < 2$ he also found (3) with $C = 1.61$, which is close to Battjes' upper limit of 1.7.

Fig. 1 shows a plot of $Ru_{2\%}/H_s$ versus the breaker parameter ξ_p , but only for plunging waves. Eq. (3), with the limits of $C = 1.3$ and 1.7, is shown in this figure, together with a set of mainly large-scale test results. The often-

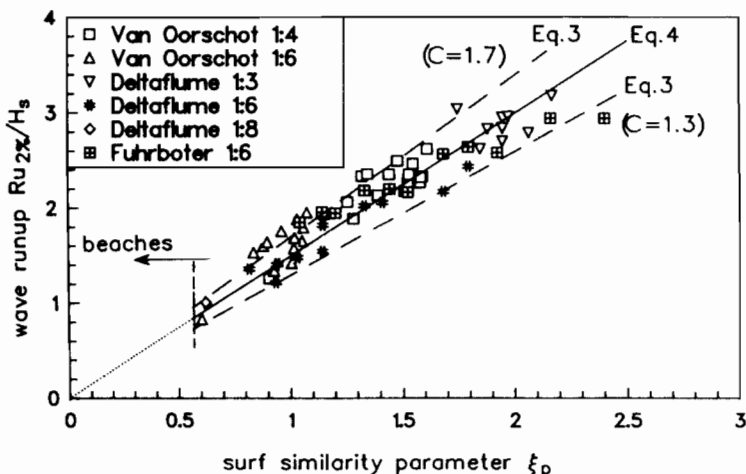


FIG. 1. Wave Runup on Smooth Slopes, Mainly Based on Large-Scale Measurements

cited results of Van Oorschot and d'Angremond (1968) are given for slopes of 1:4 and 1:6. The influence of spectral shape on runup, as described by them, is within the scatter of the other results.

All other data in Fig. 1 have been obtained in large wave flumes in the past 10 years. One set of data has been obtained in the large wave flume of Hannover University (Führbötter et al. 1989), and the other sets in the large Delta flume of Delft Hydraulics (flumes respectively 324 m and 230 m long, 5 m wide, and 7 m deep). The maximum significant wave height generated during the latter tests amounted to 1.7 m. Runup was measured on a 1:3 smooth slope consisting of a (prototype) placed block revetment, a 1:6 smooth concrete slope, and a 1:8 slope consisting of clay with grass. Führbötter et al. (1989) applied a 1:6 smooth asphalt slope and significant wave heights of as much as 1.2 m.

All data are nicely situated around a straight line through the origin, confirming the linear relationship of (3). Based on the large-scale and reliable data in Fig. 1, wave runup on gentle smooth slopes of coastal structures can be described by a linear curve through the data points as follows:

$$\frac{Ru_{2\%}}{H_s} = 1.5\xi_p \dots\dots\dots (4)$$

Eq. (4) is valid for plunging waves with $0.5 < \xi_p < 2$. The standard deviation of $Ru_{2\%}/H_s$ around this equation is $\sigma = 0.18$, which means that the 90% confidence levels are given by + or -0.3. For lower values of ξ_p (beaches), (4) may not be valid any longer [e.g., Holman (1986)].

COMPARISON OF WAVE RUNUP ON SMOOTH AND ROCK SLOPES

In Fig. 2, data for the aforementioned smooth slopes are repeated, including (4). Two data sets have been added to the figure. The data of Ahrens (1981) were added, described in the report of Ahrens and Titus (J. P. Ahrens and M. Titus, Coastal Engineering Research Center, unpublished report, "Lab Data Report: Irregular Wave Runup on Plane, Smooth Slopes," 1981). The 2% values of the runup were derived from time signals with a length

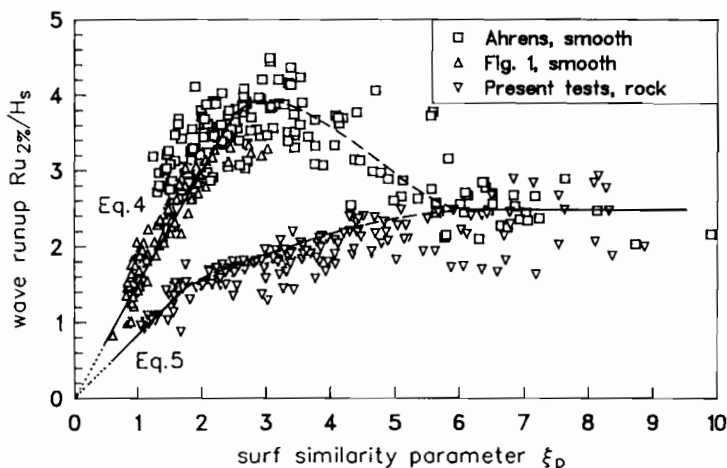


FIG. 2. Comparison of wave Runup on Smooth and Rock Slopes

of 256 s, that is from 100–200 waves. The data in Fig. 2 show a large scatter, mainly caused by the short duration of the wave and runup record. The data of the significant runup showed a much smaller scatter. Furthermore, the data of Ahrens in Fig. 2 were limited to wave steepnesses $s_p \geq 0.01$. Smaller values of s_p caused even much larger scatter.

Fig. 2 shows the general trend of runup on smooth slopes: a linear relationship with ξ_p up to $\xi_p = 2$ [(4)], a maximum runup between $\xi_p = 2$ to 4, and decreasing runup with ξ_p increasing from 4.

The second data set added to Fig. 2 gives all data of runup on rock slopes that will be treated further in this paper. The data consist of runup tests on rock slopes with $\cot \alpha = 1.5 - 4$, including impermeable (revetments) and permeable (breakwaters) structures. The general trend can be described by a linear relationship with ξ_p up to $\xi_p = 2$; increasing runup for increasing ξ_p between $\xi_p = 2$ and 6 and constant runup for $\xi_p > 6$, at approximately the same level as for smooth slopes.

A few conclusions can be drawn from a comparison of the data on smooth and rock slopes in Fig. 2. First, for large ξ_p values (i.e., nicely surging waves) little difference is present between smooth and rock slopes. Secondly, between $\xi_p = 2$ and 6 the difference in runup is large. This has also been mentioned by researchers describing monochromatic wave tests. Finally, for $\xi_p < 2$ in both cases a linear relationship is found. The average curve for rock slopes is given by

$$\frac{Ru_{2\sigma}}{H_s} = 0.83\xi_p \quad \text{for } 0.5 < \xi_p < 2 \dots\dots\dots (5)$$

Battjes (1974) gave a table of reduction factors for runup on slopes different from a smooth slope. This table is also present in the *Shore Protection Manual* (1984, Table 7.2, p. 7–32), which is widely used. Comparison of (4) and (5) indeed gives a constant reduction factor, which amounts to 0.55, though restricted to the range of $0.5 < \xi_p < 2$. The reduction factor for rock slopes given by Battjes (1974), based on monochromatic wave tests, amounted to 0.5 to 0.6.

It should be noted that, based on Fig. 2, reduction factors for runup on rough slopes as given by Battjes (1974) and SPM (*Shore* 1984) are only valid for relatively gentle slopes with $\xi_p < 2$.

TEST SETUP AND PROGRAM FOR ROCK SLOPES

An extensive series of model tests have been performed at Delft Hydraulics on the stability of rock slopes under wave attack. Results have been described briefly by Van der Meer (1987, 1988a) and the complete analysis, test results and program have been presented by Van der Meer (1988b). During all the stability tests wave runup was simultaneously measured.

All tests were conducted in a 1.0-m-wide, 1.2-m-deep, and 50-m-long wave flume, with the test section installed about 44 m from the random wave generator. A system was applied to measure and compensate for reflected waves at the wave board developed by Delft Hydraulics. With this system, standing waves and basin resonance were avoided. The incident significant wave height was measured with the structure in the flume. Two wave gauges were applied to assess the incident and reflected wave spectra.

Crushed rock was used for the armor layer, with a mass of $M_{50} = 0.123$ kg and a layer thickness of 0.08 m. The sieve analysis curves were straight lines on a log-linear plot. Two gradings were used: $D_{85}/D_{15} = 1.25$ (uniform

rock) and 2.25 (riprap), where D_{85} and D_{15} are the sieve diameters passed by 85% and 15% of the rocks by mass, respectively.

The tested cross sections are shown in Fig 3. When an impermeable core was tested, a 0.02-m-thick filter layer was placed between the armor layer and the impermeable slope (concrete in the model) [see Fig. 3(a)]. When a permeable core was tested, the armor layer was directly placed on the core, having an average diameter 3.2 times smaller than the armor layer [see Fig. 3(c)]. Also, a homogeneous structure has been tested [see Fig. 3(d)]. Note that permeability in this paper describes the structure underneath the armor layer, not the armor layer itself.

A capacitance wire was stretched along the slope, just above the armor layer, to measure the runup (and run-down). The first part of the stability test consisted of 1,000 waves, a second part of another 2,000 waves. Runup was measured only during the first part, for about 800–900 waves. The maximum value for each runup was established by an up-crossing method, and an exceedance curve was drawn for each test, taking the total number of runup heights as a reference.

Table 1 presents a general overview of the test program. Impermeable slopes were tested with $\cot \alpha = 2, 3, 4,$ and 6, with riprap and uniform rock and with various spectral shapes. Permeable slopes were tested with uniform rock and $\cot \alpha = 1.5, 2,$ and 3. A slope of $\cot \alpha = 2$ was also tested for both a homogeneous structure and a permeable structure with a 1:30 sloping foreshore in front of the structure (causing depth limited waves). Except for these latter tests, all tests were performed without a foreshore and with a water depth of 0.8 m.

About 20 tests were performed on each structure (defined by slope angle, permeability, stone shape, spectral shape, and presence of a foreshore). Each series of 20 tests contained four wave periods with about five different wave heights for each wave period. A large range of wave-height-period was tested in this way. The total number of tests amounted to about 230.

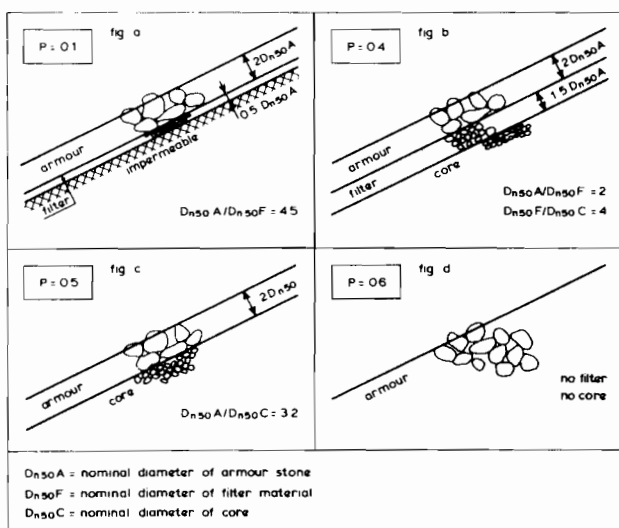


FIG. 3. Cross Sections and Notional Permeability Factor P

TABLE 1. Test Program of Wave Runup on Rock Slopes

Slope angle cot α (1)	Grading D_{85}/D_{15} (2)	Spectral Shape (3)	Core Permeability (4)	Number of tests (5)	Range $H_s/\Delta D_{r50}$ (6)	Range s_m (7)
2	2.25	PM	None	19	0.8–1.6	0.005–0.016
3	2.25	PM	None	20	1.2–2.3	0.006–0.024
4	2.25	PM	None	21	1.2–3.3	0.005–0.059
6	2.25	PM	None	26	1.2–4.4	0.004–0.063
3	1.25	PM	None	21	1.4–2.9	0.006–0.038
4	1.25	PM	None	20	1.2–3.4	0.005–0.059
3	2.25	Narrow	None	19	1.0–2.8	0.004–0.054
3	2.25	Wide	None	20	1.0–2.4	0.004–0.043
3	1.25	PM	Permeable	19	1.6–3.2	0.008–0.060
2	1.25	PM	Permeable	20	1.5–2.8	0.007–0.056
1.5	1.25	PM	Permeable	21	1.5–2.6	0.008–0.050
2	1.25	PM	Homogeneous	16	1.8–3.2	0.008–0.059
2 ^a	1.25	PM	Permeable	16	1.6–2.5	0.014–0.031

^aForeshore 1:30.

The analysis of runup tests on rock slopes will now be treated in three parts. First, a qualitative analysis is described that presents the influence of various parameters on runup. These parameters are the wave height, wave period, slope angle, structure permeability, spectral shape, and water depth. Based on this qualitative analysis two ways are followed to establish runup formulas. First, various runup levels are described in a simple way as a function of ξ_m . Finally, the runup distribution is described by means of formulas. A runup distribution gives more information than only a few levels, but is more complicated to work with as well.

QUALITATIVE ANALYSIS OF RESULTS

Various runup levels were assessed from the runup distribution. The actual analysis was based on the following levels: maximum (of about 700–800), 2%, significant (average of the highest one-third of the runup heights), and the mean level. Here, the presentation in graphs will be limited mainly to the 2% runup level. Graphs of $Ru_{2\%}/H_s$ versus the surf similarity or breaker parameter ξ_m or ξ_p show the influence of wave height, wave period, and slope angle. This kind of graph is applied for qualitative analysis of the other governing variables, such as stone shape, structure permeability, spectral shape, and presence of a foreshore (depth limited waves).

Comparison with Ahrens and Heimbaugh (1988)

Most data are given in Fig. 4, divided into a set with an impermeable core [Fig. 3(a)] and a set with a permeable core [Fig. 3(c)]. The data of Ahrens and Heimbaugh (1988) are added to this figure. They measured the approximate upper limit of irregular wave runup on riprap slopes, comparable with the tests on an impermeable core. The upper limit of the runup was obtained by visual observations during 256 s. However, this short time limits the number of runup heights to about 100–200, which means that the maximum runup will probably be close to the 2% runup level of the present tests.

From Fig. 4, it can be concluded that the data of Ahrens and Heimbaugh (1988) agree well with the impermeable core data, although they display larger scatter.

Influence of Wave Height, Wave Period, and Slope Angle

The surf similarity parameter includes the wave height, period, and slope angle. All three variables were varied for the same slope permeability and spectral shape. Fig. 5 presents the 2% and significant runup levels for the tests on an impermeable core [Fig. 3(a)]. As all data lay nicely on one and the same curve for each runup level, the surf similarity parameter proves to be a good parameter for describing the combined effects of wave height, period, and slope angle.

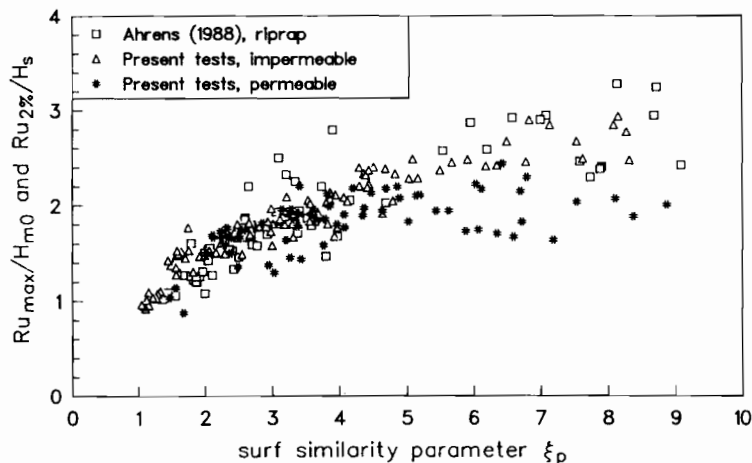


FIG. 4. Comparison of Wave Runup Data on Rock Slopes with Ahrens and Heimbaugh (1988)

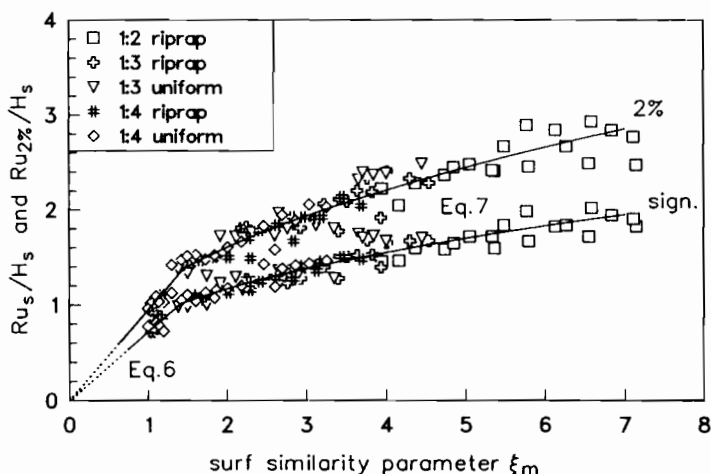


FIG. 5. Wave Runup on Impermeable Rock Revetments

Fig. 5 includes both the tests on uniform rock with $D_{85}/D_{15} = 1.25$ and on riprap with $D_{85}/D_{15} = 2.25$. Comparison of the data showed no difference between both gradings.

Influence of Structure Permeability

Various slope angles were tested with an impermeable core [Fig. 3(a)], a permeable core [Fig. 3(c)] and a homogeneous structure [Fig. 3(d), only large rock]. Fig. 6 shows the results. Up to $\xi_m = 3$, the runup values are more or less the same for all three permeabilities. For larger values of ξ_m , the permeable and homogeneous structures reach more or less a same constant level of $Ru_{2\%}/H_s$ around 2, while the impermeable core data still show increasing runup with increasing ξ_m , up to $\xi_m = 7$.

This latter phenomenon can be explained in physical terms by the difference in water motion on and in the structure. Surging waves for $\xi_m > 2$ or 3 surge slowly up the slope without energy loss due to breaking. With permeable structures the runup sinks into the structure, while for an impermeable structure all water running up remains completely in and on the armor layer.

Influence of Spectral Shape

The one-dimensional spectrum is characterized by its length scale, time scale, and shape. The length scale is often the significant wave height and the time scale a characteristic period, say, the mean period or the peak period. Various parameters have been developed to describe the shape or width of the spectrum. Well-known parameters are ε (Cartwright and Longuet-Higgins 1956) and Q_p (Goda 1970). Other parameters were based on the wave signal and described the groupiness of the waves in the time domain. The mean length, $\bar{j}_1(H)$, and the mean total length, $\bar{j}_2(H)$, of the wave groups (Goda 1970) and the groupiness factor, GF (Funke and Mansard 1980), are some of these parameters. The parameter κ can be computed both in the time and frequency domains. The parameter in the frequency domain, κ_f , was defined by Rice (1945) and that in the time domain, $\kappa_{AA,t}$

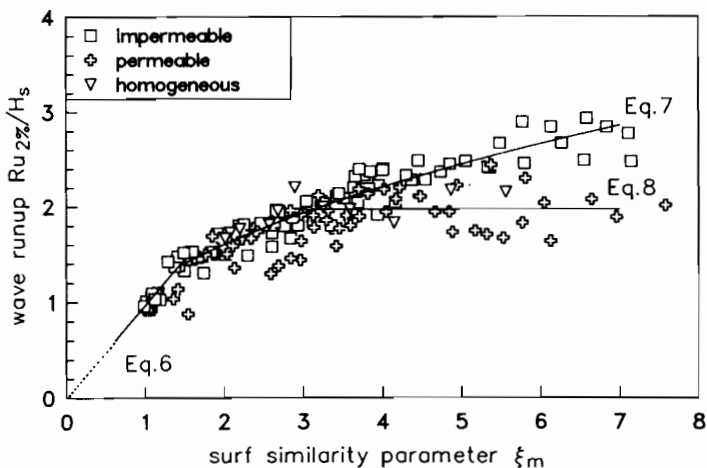


FIG. 6. Influence of Permeability of Structure on Wave Runup

by Arhan and Erzaty (1978). See for the κ -parameter also Van der Meer (1988b).

Most tests were performed with a Pierson-Moskowitz (PM) spectrum. About 20 tests, however, were performed with a (very) wide spectrum and an additional number of 20 tests with a very narrow spectrum. The slope angle was $\cot \alpha = 3$ and the structure was impermeable. The average values of all these spectral parameters are shown in Table 2 for the three spectral shapes. Spectral shapes are given in Van der Meer (1988b).

Fig. 7 shows an example of three runup distributions, one for each spectral shape. The wave height for all three tests amounted to $H_s = 0.12$ m and the mean wave period to $T_m = 1.8$ s. The runup curves for the PM and wide spectrum are fairly straight. The curve corresponding to the narrow

TABLE 2. Spectral Parameters for Three Spectral Shapes

Parameter (1)	Narrow spectrum (2)	PM spectrum (3)	Wide spectrum (4)
$\epsilon_{s,t}$	0.10	0.39	0.59
Q_p	13.4	2.6	1.5
GF	0.99	0.77	0.72
$\bar{j}_1(\bar{H})$	5.4	2.6	2.2
$\bar{j}_2(\bar{H})$	12.6	5.6	4.8
$\bar{j}_1(H_s)$	2.95	1.54	1.35
$\bar{j}_2(H_s)$	22.0	11.3	10.3
κ_f	0.91	0.48	0.25
$\kappa_{AA,t}$	0.92	0.58	0.39
T_p/T_m	1.01	1.15	1.42

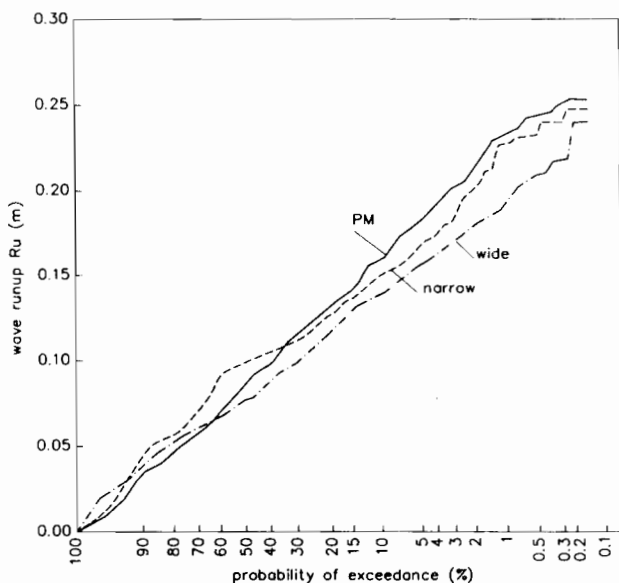


FIG. 7. Wave Runup Distributions for Three Spectral Shapes

spectrum is less straight. This may probably be caused by the fact that the narrow spectrum was very narrow, maybe out of the practical range in nature.

Figs. 8 and 9 show $Ru_{2\%}/H_s$ versus ξ_m , respectively ξ_p . In Fig. 8, the data of the PM and the wide spectrum agree well, whereas the data of the narrow spectrum are lower in the lower ξ_m region. In Fig. 9 the data of the PM and the narrow spectrum agree well, but the data of the wide spectrum are consequently lower. As the narrow spectrum is less realistic preference is given to the best agreement between the PM and the wide spectrum. This means that the use of the mean period T_m in ξ_m gives similar results for different (not too narrow) spectral shapes and that the use of a spectral width parameter is not required to describe runup.

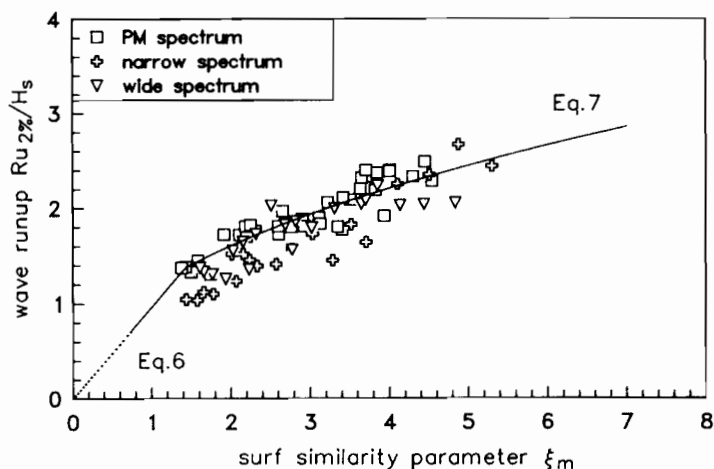


FIG. 8. Influence of Spectral Shape on Runup as Function of ξ_m

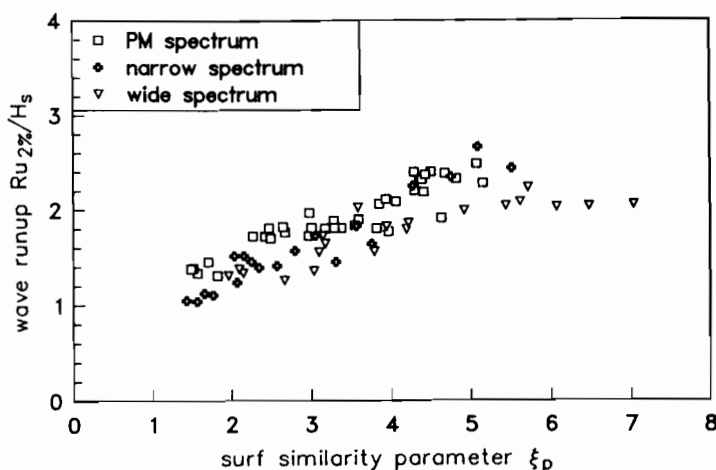


FIG. 9. Influence of Spectral Shape on Runup as Function of ξ_p

Influence of Water Depth

Most tests were performed with a water depth of 0.8 m and without a foreshore. About 20 tests were performed with a 15-m-long, 1:30 foreshore in front of the structure. The water depth at the structure was 0.4 m in half of the test series (only high waves breaking on the foreshore) and 0.2 m in the other half (severe wave breaking in most tests). The wave heights on the horizontal bottom (0.9 m or 0.7 m deep) were Rayleigh distributed, but this distribution changed due to the influence of the foreshore. Calibration tests without the structure in the flume gave the wave height distribution at the location of the toe of the structure. These wave heights at the toe of the structure were used in the further analysis.

Fig. 10 shows a runup distribution curve for each of the three water depths applied. The graph for the largest depth $h = 0.8$ m shows a distribution close to the Rayleigh distribution (a straight line in this plot). For a water depth of $h = 0.4$ m runup heights are Rayleigh distributed up to the 20% value after which the distribution becomes truncated. The lowest water depth of $h = 0.2$ m with severe wave breaking seaward of the toe of the structure shows higher runup for the lowest part of the distribution and, again, a truncation for the higher part.

Fig. 11 presents the data in a runup versus ξ_m plot. Both the 2% and the mean runup levels are given. Most of the 2% values are lower for both $h = 0.4$ m and 0.2 m compared with $h = 0.8$ m. The mean runup for 0.8 m and 0.4 m is the same, whereas the 0.2 m water depth clearly gives higher runup. This trend is similar to that as found in Fig. 10. It can be concluded that wave breaking on a foreshore results in a truncation in the runup

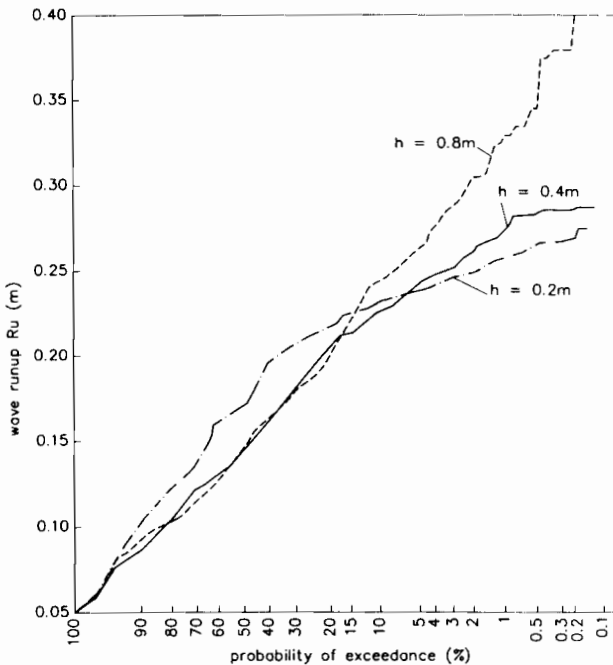


FIG. 10. Wave Runup Distributions for Various Water Depths in Front of Structure

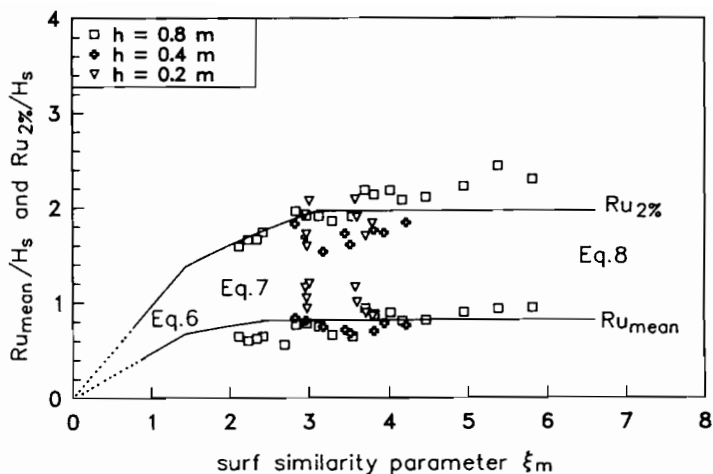


FIG. 11. Influence of Water Depth in Front of Structure on Wave Runup

distribution, which mainly results in lower maximum runup heights, although sometimes higher mean runup heights may occur.

RUNUP FORMULAS AS FUNCTION OF ξ_m

The surf similarity parameter ξ_m (or ξ_p) has proved to be a good parameter to describe wave runup on smooth and rock slopes. It includes both slope angle and wave steepness. The permeability of the structure has only influence on runup in the high ξ_m region, where runup reaches a maximum for permeable structures. A wide spectrum and a PM spectrum give similar runup when using the mean period for comparison. In that case, the influence of the spectral shape is negligible.

A first analysis of the wave runup heights was performed in order to describe various runup levels in a simple way as a function of the surf similarity parameter ξ_m . The runup levels chosen were: the maximum (of about 800 waves, yielding 0.13%), 1, 2, 5, 10%, the significant level (average of the highest one-third of the runup heights) and the mean runup level. In agreement with the results on smooth slopes, described in the first part of this paper, a linear relationship between relative runup and ξ_m was taken for small ξ_m values, say, smaller than about 1.5. For larger ξ_m values the runup grows more slowly (see Fig. 5). That relationship can be described simply by a power function. Some authors used one formulation to describe runup as a function of ξ_m . The combination of linear and power curves was mainly chosen to provide a direct comparison with breaking waves on smooth slopes. A maximum is reached at a certain ξ_m value for permeable slopes (see Fig. 6). The relationships are given by

$$\frac{Ru_x}{H_s} = a\xi_m \quad \text{for } \xi_m \leq 1.5 \quad \dots\dots\dots (6)$$

$$\frac{Ru_x}{H_s} = b\xi_m^c \quad \text{for } \xi_m \geq 1.5 \quad \dots\dots\dots (7)$$

where the index x is a certain runup level. The runup for permeable slopes is limited to a maximum

$$\frac{Ru_x}{H_s} = d \dots\dots\dots (8)$$

Revetments like that in Fig. 3(a) can be considered impermeable, all other structures in Fig. 3 are permeable. Values for the coefficients a , b , c , and d in (6)–(8) are given in Table 3. The equations are also shown in Figs. 5, 6, 8, and 11, together with test results.

Eqs. (6)–(8) are valid for relatively deep water in front of the structure, where the wave height distribution is close to a Rayleigh distribution. Wave breaking on a foreshore results in a truncation in the runup distribution, which mainly results in lower maximum runup heights, although sometimes higher mean runup heights may occur (see Figs. 10 and 11). As the number of tests under depth-limited conditions was small, it was not possible to include the effect of depth limitation into (6)–(8). Use of (6)–(8), however, gives a conservative estimation of high runup values under depth-limited conditions.

The reliability of (6)–(8) can be described by assuming coefficients a , b , and d to be stochastic variables with a normal distribution. The variation coefficients (standard deviation/mean) for these coefficients are 7% for impermeable structures and 12% for permeable structures. Based on these variation coefficients confidence bands can be calculated.

WAVE RUNUP AS WEIBULL DISTRIBUTION

The previous analysis on runup was limited to a number of runup levels. The complete runup distribution provides more information than only a number of runup levels. Therefore, a second analysis of the runup was based on the runup distributions, as shown in Figs. 7–10.

First, the runup distributions of all tests were analyzed and compared with the Rayleigh distribution. This analysis showed that a large number of runup distributions were close to a Rayleigh distribution. But a considerable number of tests showed deviations, influenced by wave steepness, slope angle, and spectral shape. Therefore, the Rayleigh distribution was not taken for the description of runup.

The Weibull distribution chosen herein is fairly simple and also includes the Rayleigh distribution. The Weibull distribution can be written as

$$p = p(Ru > Ru_p) = \exp \left[- \left(\frac{Ru_p}{e} \right)^f \right] \dots\dots\dots (9)$$

TABLE 3. Coefficients for Runup Levels to be Used in Eqs. (6)–(8)

Level (%) (1)	a (2)	b (3)	c (4)	d (5)
0.13	1.12	1.34	0.55	2.58
1	1.01	1.24	0.48	2.15
2	0.96	1.17	0.46	1.97
5	0.86	1.05	0.44	1.68
10	0.77	0.94	0.42	1.45
Significant	0.72	0.88	0.41	1.35
Mean	0.47	0.60	0.34	0.82

which can be rewritten as

$$Ru_p = e(-\ln p)^{1/f} \dots\dots\dots (10)$$

where p = probability of exceedance (between 0 and 1); Ru_p = runup level exceeded by $p \cdot 100$ per cent of the runup levels; e = scale parameter; f = shape parameter.

The scale parameter e is independent of the shape parameter and is equal to the runup level that is exceeded by $e^{-1} = 0.37$ (37%) of the runup levels. The shape parameter f defines the shape of the curve. For $f = 2$, a Rayleigh distribution is obtained (a straight line on plots as Figs. 7 and 10). For $f > 2$, the distribution becomes narrower, resulting in a downward curved distribution (see Fig. 10).

The analysis showed that the scale parameter e could not be described by ξ_m . A graph of e/H_s versus ξ_m showed different curves for different slope angles. Therefore, the slope $\cot \alpha$ and the wave steepness s_m were treated independently. The scale parameter can be described by

$$\frac{e}{H_s} = 0.4s_m^{-0.25} \cot \alpha^{-0.2} \dots\dots\dots (11)$$

The shape parameter is described by

$$f = 3.0\xi_m^{-0.75} \text{ for plunging waves} \dots\dots\dots (12)$$

and

$$f = 0.52P^{-0.3}\xi_m^P \sqrt{\cot \alpha} \text{ for surging waves} \dots\dots\dots (13)$$

The transition between (12) and (13) is described by a critical value for the surf similarity parameter, ξ_{mc}

$$\xi_{mc} = (5.77P^{0.3}\sqrt{\tan \alpha})^{1/(P + 0.75)} \dots\dots\dots (14)$$

For $\xi_m < \xi_{mc}$, (12) should be used and for $\xi_m > \xi_{mc}$, (13).

The factor P defines the permeability of the structure [see van der Meer (1988b)] and Fig. 3. P is called the notional permeability factor, and describes the permeability of the structure underneath the armour layer. The permeability depends on the size of underlayers, filter layers, and core. The lower limit of $P = 0.1$ is an armor layer with a thickness of two diameters on an impermeable core (sand or clay) and with only a thin filter layer (a revetment) [see Fig. 3(a)]. The upper limit of $P = 0.6$ is given by a homogeneous structure, which consists only of armor stones [see Fig. 3(d)]. Breakwaters with a two-diameter-thick armor layer, one or more underlayers and a core have P -values on the order of $P = 0.4$ – 0.5 [see Figs. 3(b) and 3(c)].

Eqs. (12)–(14) have the same limitations with regard to depth limitation as (6)–(8). Furthermore, (12)–(14) are only applicable for slopes with $\cot \alpha \geq 2$. For steeper slopes, the runup distribution on a 1:2 slope may give a first estimation.

The reliability of (12)–(14) can be described by assuming e as a stochastic variable with a normal distribution. The variation coefficient of e is 6% for $P < 0.4$ and 9% for $P \geq 0.4$. Confidence bands can be calculated by means of these variation coefficients.

A simple computer program can be made, based on (6)–(8) and (12)–

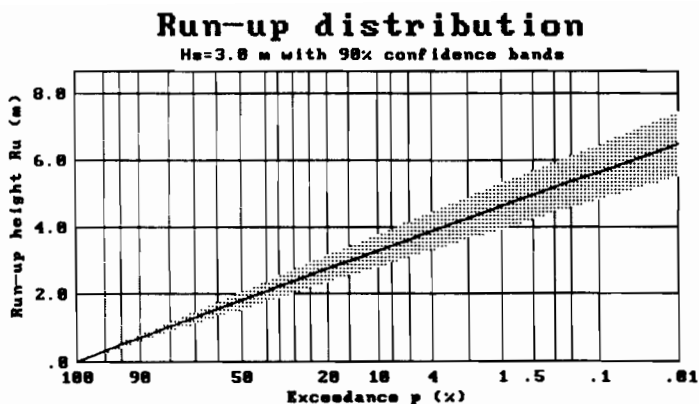


FIG. 12. Example of Calculated Runup Distribution with Confidence Bands for $\cot \alpha = 3$, $H_s = 3$ m, $T_m = 7$ s, and $P = 0.4$

(14). Such a program can generate a table with runup values, a graph of runup versus the wave steepness, or a graph of the runup distribution. Confidence bands can be taken into account. An example is shown in Fig. 12, which gives a runup distribution with confidence bands on a slope with $\cot \alpha = 3$.

SUMMARY AND CONCLUSIONS

The first part of the paper described runup on smooth slopes. A design formula for smooth gentle slopes was established, based on large-scale tests on various slopes [(5)]. A comparison between runup on smooth and rock slopes resulted in two conclusions. First, a reduction factor for runup on rough slopes based on formulas for smooth slopes, as suggested by the SPM, is only applicable for gentle slopes with $\xi_p < 2$ (see Fig. 2). The second conclusion is that for $\xi_p > 6$ smooth and rock slopes yield similar runup levels.

In the second part of the paper, extensive runup measurements on rock slopes were treated. An analysis resulted in two sets of runup equations. One set presents various relative runup levels as a function of the surf similarity parameter ξ_m . The other set describes the runup distribution as a Weibull distribution. Both sets of equations have been implemented in a computer program that can generate a table of runup values or runup graphs.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

- a, b, c, d, e, f = coefficients;
- D_{15} = diameter determined by 15% value on sieve curve;
- D_{85} = diameter determined by 85% value on sieve curve;
- g = gravitational acceleration;
- H_s = significant wave height;
- h = water depth in front of structure;
- M_{50} = average mass of rock grading, determined by 50% value on mass distribution curve;

- P = notional permeability factor (between 0.1 and 0.6, see Fig. 3);
 p = probability of exceedance (between 0 and 1);
 Ru_p = wave runup level (index p gives level: 0.13, 1, 2, 5, 10%, significant and mean);
 s_m = wave steepness, $s_m = 2\pi H_s/gT_m^2$;
 s_p = wave steepness, $s_p = 2\pi H_s/gT_p^2$;
 T_m = mean wave period;
 T_p = spectral peak period;
 α = slope angle of structure;
 ξ_m = surf similarity parameter, $\xi_m = \tan \alpha/\sqrt{s_m}$;
 ξ_p = surf similarity parameter, $\xi_p = \tan \alpha/\sqrt{s_p}$; and
 σ = standard deviation.